NEET | IIT-JEE | FOUNDATION
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JEE Advanced : Paper-II (2019)

IMPORTANT INSTRUCTIONS

- This section contains EIGHT (08) questions.
- Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s)
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+ 4	If only (all) the correct option(s) is (are) chosen.
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen.
Partial Marks	:	+2	If three or more options are correct but ONLY two options are chosen and both of which are correct.
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:	–1	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;

choosing ONLY (A) and (B) will get +2 marks;

choosing ONLY (A) and (D) will get +2 marks;

choosing ONLY (B) and (D) will get +2 marks;

choosing ONLY (A) will get +1 marks;

choosing ONLY (B) will get +1 marks;

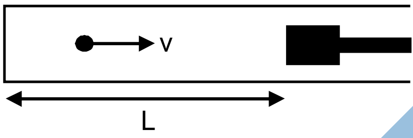
choosing ONLY (D) will get +1 marks;

choosing no option (i.e. the question is unanswered) will get 0 marks, and

choosing any other combination of options will get –1 mark.

PART-A: PHYSICS

1. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L} v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct ?



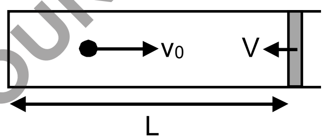
(1) The rate at which the particle strikes the piston is v/L

(2*) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to L_0

(3*) After each collision with the piston, the particle speed increases by 2V

(4) If the piston moves inward by dL, the particle speed increases by 2v

Sol. (1) average rate of collision = $\frac{2L}{v}$



(2) speed of particle after collision = $2V + v_0$

change in speed = $(2V + v_0) - v_0$

after each collision = 2V

no. of collision per unit time (frequency) = $\frac{v}{2L}$

change in speed in dt. time = 2V × number of collision in dt time

$\Rightarrow dv = 2V\left(\frac{v}{2L}\right).\frac{dL}{V}$

$dv = \frac{vdL}{L}$

Now, $dv = -\frac{vdL}{L}$ {as L decrease}

$\int_{v_0}^{v} \frac{dv}{v} = -\int_{v_0}^{L_0/2} \frac{dL}{L}$

$\Rightarrow [\ln v]_{v_0}^{v} = -[\ln L]_{L}^{L_0/2}$

$\Rightarrow v = 2v_0$

$\Rightarrow KE_{L_0} = \frac{1}{2}mv_0^2 \qquad \frac{KE_{L_0/2}}{KE_0} = 4$

$KE_{L_0/2} = \frac{1}{2}m(2v_0)^2$

or

$(dt)\left(\frac{v}{2x}\right)\frac{2mv}{dt} = F \quad = F$

$F = \frac{mv^2}{x}$

$-mv\frac{dv}{dx} = \frac{mv^2}{x}$

$-\frac{dv}{v} = \frac{dx}{x}$

$\ln\frac{v_2}{v_1} = \ln\frac{x_1}{x_2}$

vx = constant $\Rightarrow$ on decreasing length to half K.E. becomes 1/4 vdx + xdv = 0

2. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical ? [g is the acceleration due to gravity]

(1*) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

(2*) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

(3) The angular acceleration of the rod will be $\frac{2g}{L}$

(4*) The radial acceleration of the rod's center of mass will be $\frac{3g}{4}$

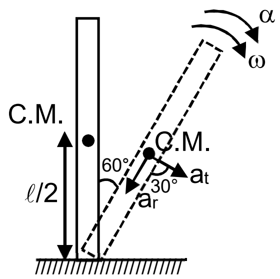
Sol. We can treat contact point as hinged,

Applying work energy theorem

$W_g = \Delta KE$

$mg\frac{\ell}{4} = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$

$\omega = \sqrt{\frac{3g}{2\ell}}$



radial acceleration of CM of rod = $\left(\frac{\ell}{2}\right)\omega^2 = \frac{3g}{4}$

Using $\tau = I\alpha$ about contact point

$$\frac{mg\ell}{2}\sin 60^\circ = \frac{m\ell^2}{3}\alpha$$

$$\Rightarrow \alpha = \frac{3\sqrt{3}}{4\ell}g$$

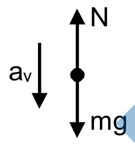
Net vertical acceleration of CM of rod

$a_v = a_r \cos 60° + a_t \cos 30°$

$$= \left(\frac{3g}{4}\right)\left(\frac{1}{2}\right) + \left(\alpha\frac{\ell}{2}\right)\cos 30^\circ$$

$$\left(\frac{3g}{4}\right)\left(\frac{1}{2}\right) + \left(\alpha\frac{\ell}{2}\right)\cos 30^\circ$$

$$= \frac{3g}{4} + \frac{3\sqrt{3}\,g}{4\ell}\left(\frac{\ell}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)o$$



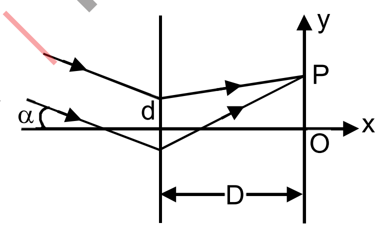
$$= \frac{3g}{8} + \frac{9g}{16} = \frac{15}{16}g$$

Applying F_{net} = ma in vertical direction on rod as system

$$mg - N = ma_v = m\left(\frac{15}{16}g\right)$$

$$\Rightarrow N = \frac{mg}{16}$$

3. In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1m. A parallel beam of light of wavelength 600nm is incident on the slits at angle α as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11.0 mm. Which of the following statement(s) is/are correct ?



(1*) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point P

(2) Fringe spacing depends on α.

(3) For α=0, there will be constructive interference at point P.

(4) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point O.

Sol. (1) $\Delta x = d \sin \alpha$

$= d\alpha$ (as α is very small) $\alpha = \frac{0.36}{180}$ = (2×10^{-3}) rad

$$\frac{\Delta x}{\lambda} = \frac{(3\times10^{-4})(2\times10^{-3})}{6\times10^{-7}} = 1$$

So constructive interference

(2) $\beta = \frac{D\lambda}{d}$

(3) $\Delta x_p = d\alpha + \frac{dy}{D}$

$= 3 \times 10^{-4} (2 \times 10^{-3} + 11 \times 10^{-3})$

$= 39 \times 10^{-7}$

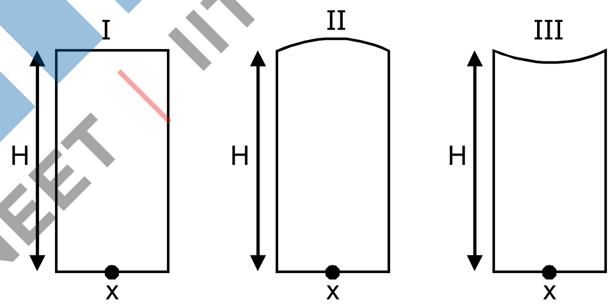
$\frac{\Delta x_p}{\lambda} = \frac{39\times10^{-7}}{6\times10^{-7}}$ = 6.5 so destructive

(4) $\Delta x_p = \frac{dy}{D}$ = $(3 \times 10^{-4}) \times 11 \times 10^{-3}$

$= 33 \times 10^{-7}$

$\frac{\Delta x_p}{\lambda} = \frac{33\times10^{-7}}{6\times10^{-7}} = 5.5 \Rightarrow$ destructive

4. Three glass cylinders of equal height H = 30 cm and same refractive index n = 1.5 are placed on a horizontal surfaces shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same (R = 3m). If H_1, H_2 and H_3 are the apparent depths of a point X on the bottom of the three cylinders, respectively, the correct statement(s) is/are



(1) $H_3 > H_1$

(2*) $H_2 > H_1$

(3*) $H_2 > H_3$

(4) $0.8\text{ cm} < (H_2 - H_1) < 0.9\text{cm}$

Sol. $H_1 = \frac{2H}{3} = \frac{2}{3}\times\frac{3}{10} = \frac{1}{5}\text{m}$

for 2[nd]

$$\frac{1}{v}+\frac{3}{2H}=\frac{-1}{2(-3)}$$

$$\frac{1}{v}=\frac{1}{6}-\frac{10}{2}=\frac{1}{6}-\frac{30}{6}=\frac{-29}{6}$$

$$H_2=\frac{6}{29}>H_1$$

For 3rd

$$\frac{1}{v}+\frac{3}{2H}=\frac{-1}{2(3)}$$

$$\frac{1}{v}=\frac{-1}{6}-5=\frac{-31}{6}$$

$$H_3=\frac{6}{31}$$

So $H_3 < H_1 < H_2$ and $(H_2 - H_1) = \frac{6}{29}-\frac{6}{31}$ = 0.68 cm

5. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state n = 1 to the state n = 4. Immediately after that the electron jumps to n = m state by emitting a photon of wavelength λ_e. Let the change in momentum of atom due to the absorption and the emission are Δp_a and Δp_e, respectively. If $\lambda_a / \lambda_e = \frac{1}{5}$, Which of the option(s) is/are correct ?

[Use hc = 1242 eV nm; 1 nm = 10^{-9} m, h and c are Planck's constant and speed of light, respectively]

(1*) The ratio of kinetic energy of the electron in the state n = m to the state n = 1 is $\frac{1}{4}$

(2*) m = 2

(3) λ_e = 418nm

(4) $\Delta p_a / \Delta p_e = \frac{1}{2}$

Sol. $\frac{hc}{\lambda_a}=13.6\left[\frac{1}{2}-\frac{1}{4^2}\right]$ ......(i)

$\frac{hc}{\lambda_e}=13.6\left[\frac{1}{m^2}-\frac{1}{4^2}\right]$ ......(ii)

(ii) / (i), we get

$$\frac{\lambda_a}{\lambda_e}=\frac{\left[\frac{1}{m^2}-\frac{1}{16}\right]}{\left[1-\frac{1}{16}\right]}=\frac{1}{5}$$

$$\Rightarrow \frac{1}{m^2} - \frac{1}{16} = \frac{15}{16} \times \frac{1}{5} \qquad \Rightarrow \frac{1}{m^2} - \frac{1}{16} = \frac{3}{16}$$

$$\Rightarrow \frac{1}{m^2} = \frac{3}{16} + \frac{1}{16} \qquad \Rightarrow m = 2$$

from (ii)

$$\frac{hc}{\lambda_e} = 13.6\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = 13.6 \times \frac{3}{16} eV \qquad \Rightarrow \lambda_e = \frac{12400 \times 16}{13.6 \times 3} Å$$

$$\Rightarrow \lambda_e \approx 4862Å$$

we have

$$KE_n \propto \frac{z^2}{n^2}$$

$$\Rightarrow \frac{KE_2}{KE_1} = \frac{1}{4}$$

$$\Delta P_a = \frac{h}{\lambda_a}$$

$$\Delta P_e = \frac{h}{\lambda_e}$$

$$\frac{\Delta P_a}{\Delta P_e} = \frac{\lambda_e}{\lambda_a}$$

6. A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0, volume V_0 and temperature T_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are, (Give $2^{1.2} = 2.3$; $2^{3.2} = 9.2$; R is gas constant)

(1*) The work (W) done during the process is $13RT_0$

(2) The average kinetic energy of the gas mixture after compression is in between $18RT_0$ and $19RT_0$

(3*) Adiabatic constant of the gas mixture is 1.6

(4*) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$

Sol. $n_1 = 5$ moles $C_{v_1} = \frac{3R}{2}$ $P_0V_0T_0$

$n_2 = 1$ mole $C_{v_2} = \frac{5R}{2}$

$$(C_V)_m = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2} = \frac{5 \times \frac{3R}{2} + 1 \times \frac{5R}{2}}{6} = \frac{5R}{3}$$

$$\gamma_m = \frac{(c_P)_m}{(c_V)_m} = \frac{8}{5}$$

$(C_P)_m = \dfrac{5R}{3} + R = \dfrac{8R}{3}$

(1) $P_0V_0^{\gamma} = P\left(\dfrac{V_0}{4}\right)^{\gamma}$ $\Rightarrow$ P = $P_0(4)^{8/5}$ = 9.2 P_0 which is between $9P_0$ and $10P_0$

(2) Average K.E. = 5 × $\dfrac{3}{2}RT + 1 \times \dfrac{5RT}{2}$ = 10RT

To calculate T

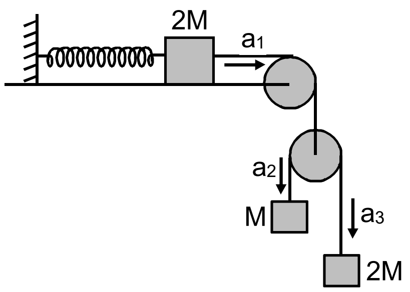
$\dfrac{P_0V_0}{T_0} = 9.2\,P_0 \times \dfrac{V_0}{4 \times T}$

So T = $\dfrac{9.2}{4}T_0$

Now average KE = 10R × 9.2 $\dfrac{T_0}{4}$ = 23RT_0

(3) $W = \dfrac{P_1V_1 - P_2V_2}{\gamma - 1} = \dfrac{P_0V_0 - 9.2P_0 \times \dfrac{V_0}{4}}{3/5}$ = –13 RT_0

7. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct ? [g is the acceleration due to gravity. Neglect friction]



(1*) $a_2 - a_1 = a_1 - a_3$

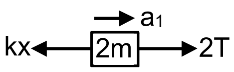
(2) $x_0 = \dfrac{4Mg}{k}$

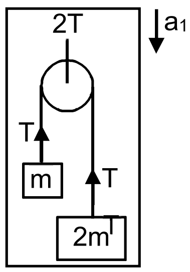
(3) When spring achieves an extension of $\dfrac{x_0}{2}$ for the first time,

the speed of the block connected to the spring is $3g\sqrt{\dfrac{M}{5k}}$

(4) At an extension of $\dfrac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\dfrac{3g}{10}$

Sol. $2T - kx = 2ma_1$





$$T = \frac{2(2m)(m)}{3m}(g-a_1) = \frac{4m}{3}(g-a_1)$$

$$\frac{8m}{3}(g-a_1) - kx = 2ma_1$$

$$\frac{8mg}{3} - \frac{8ma_1}{3} - kx = 2ma_1$$

$$\frac{8Mg}{3} - kx = \frac{14ma_1}{3}$$

$$\frac{8Mg-3kx}{14m} = a_1$$

$$a_1 = \frac{8Mg-3kx}{14m}$$

$$\frac{vdv}{dx} = \left(\frac{8Mg}{14} - \frac{3kx}{14m}\right)$$

$$\int vdv = \frac{1}{14m}\int(8Mg-3kx)dx$$

for max elongation

$$0 = \frac{1}{14m}\int_0^{x_0}(8Mg-3kx)dx$$

$$= \frac{1}{14m}\left(8Mgx_0 - \frac{3kx_0^2}{2}\right)$$

$$8Mgx_0 = \frac{3kx_0^2}{2}$$

$$x_0 = \frac{16Mg}{3k}$$

at $x = \frac{x_0}{2}$

$$\int_0^v vdv = \frac{1}{14m}\int_0^{x_0/2}(8Mg-3kx)dx$$

$$\frac{v^2}{2} = \frac{1}{14m}\left(\frac{8Mgx_0}{2} - \frac{3kx_0^2}{2\times4}\right)$$

$$v^2 = \frac{1}{7m}\left(\frac{8Mg}{2}\times\frac{16Mg}{3x} - \frac{3x}{8}\times\frac{16M^2g^2}{3x\times3x}\right)$$

$$= \frac{1}{7m}\left(\frac{64M^2g^2}{3x} - \frac{2M^2g^2}{3x}\right)$$

$$v^2 = \frac{62Mg^2}{21k}$$

For acc. $2a_1 = a_2 + a_3$ therefore

$a_2 - a_1 = a_1 - a_3$

$$a_1 = \frac{8Mg - 3kx_0/4}{14m} = \frac{8g}{14} - \frac{3kx_0}{14m \times 4} = \frac{8g}{14} - \frac{3x}{14m \times 4} \times \frac{16Mg}{3x}$$

$$= \frac{8g}{14} - \frac{4g}{14} = \frac{4g}{14} = \frac{2g}{7}$$

OR

$$\frac{8mg}{3} - \frac{8m}{3}a_1 - kx = 2ma_1$$

$$\frac{14m}{3}a_1 = -k\left[x - \frac{8mg}{3k}\right]$$

$$a_1 = -\frac{3k}{14m}\left[x - \frac{8mg}{3k}\right] \quad .....(i)$$

that means, block 2m (connected with the spring) will perform SHM about $x_1 = \frac{8mg}{3k}$

therefore,

maximum elongation in the spring $x_0 = 2x_1 = \frac{16mg}{3k}$

on comparing equation (1) with

$a = \omega^2(x - x_0)$

$$\omega = \sqrt{\frac{3k}{14m}}$$

at $\left(\frac{x_0}{2}\right)$, block will be passing through its mean position therefore at mean position

$$v_0 = A\omega = \frac{8mg}{3k}\sqrt{\frac{3k}{14m}}$$

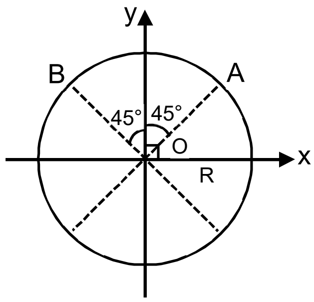
At $\frac{x_0}{4} \Rightarrow x = \frac{A}{2}$

$$\therefore a_{cc} = -\frac{A}{2}\omega^2$$

$$= -\frac{4mg}{3k}\frac{3h}{14m} = -\frac{2g}{7}$$

8. An electric dipole with dipole moment $\frac{p_0}{\sqrt{2}}(\hat{i} + \hat{j})$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are:

($\in_0$ is permittivity of free space, R >> dipole size)



(1) The magnitude of total electric field on any two points of the circle will be same

(2) Total electric field at point A is $\vec{E}_A = \sqrt{2}E_0(\hat{i}+\hat{j})$

(3*) $R = \left(\dfrac{p_0}{4\pi\epsilon_0 E_0}\right)^{1/3}$

(4*) Total electric field at point B is $\vec{E}_B = 0$

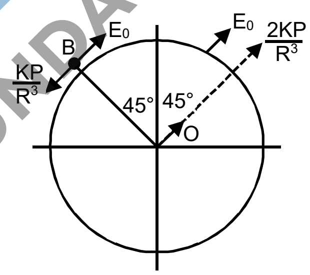
Sol. (1) $\vec{P} = \dfrac{P_0}{\sqrt{2}}(\hat{i}+\hat{j})$

E.F. at B along tangent should be zero since circle is equipotential.

So, $E_0 = \dfrac{K|\vec{P}|}{R^3}$ and $E_B = 0$

So, $R^3 = \dfrac{KP_2}{E_0} = \left(\dfrac{P_0}{4\pi\epsilon_0 E_0}\right)$

So $R = \left(\dfrac{P_0}{4\pi\epsilon_0 E_0}\right)^{1/3}$



(2) Because E_0 is uniform and due to dipole E.f. is different at different points, so magnitude of total E.F. will also be different at different points.

(3) $E_A = \dfrac{2KP}{R^3} + \dfrac{KP}{R_3} = 3\dfrac{KP}{R^3}\dfrac{P_0}{\sqrt{2}}(\hat{i}+\hat{j})$

(4) $E_B = 0$

SECTION-2 : (Maximum Marks: 18)

- This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round-off the value to Two decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+ 3	If ONLY the correct numerical value is entered
Zero Marks	:	0	In all other cases

9. An optical bench has 1.5 m long scale having four equal divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is________.

Ans. 1.39 (1.35 to 1.45)

Sol. For the given lens

u = –30 cm

v = 60 cm

and $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ on solving : f = 20 cm

also $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

on differentiation

$$\frac{df}{f^2} = \frac{dv}{v^2} + \frac{du}{u^2}$$

$$\frac{df}{f} = f\left[\frac{dv}{v^2} + \frac{du}{u^2}\right] \text{ and } \frac{df}{f} \times 100 = f\left[\frac{dv}{v^2} + \frac{du}{u^2}\right] \times 100\%$$

f = 20 cm, du = dv = $\frac{1}{4}$ cm

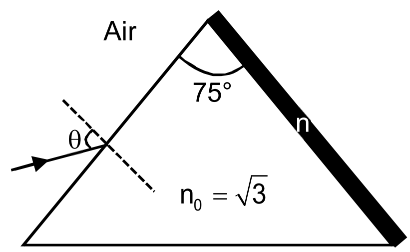
Since there are 4 divisions in 1 cm on scale

$$\therefore \frac{df}{f} \times 100 = 20\left[\frac{1/4}{(60)^2} + \frac{1/4}{(30)^2}\right] \times 100\%$$

$$= 5\left[\frac{1}{3600} + \frac{1}{900}\right] \times 100\%$$

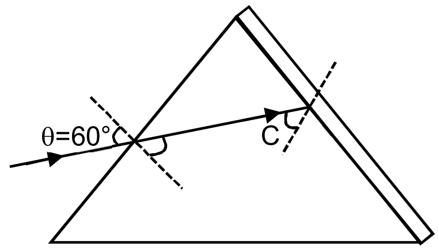
$$= 5\left[\frac{5}{36}\right]\% = \frac{25}{36}\% \approx 0.69\%$$

10. A monochromatic light is incident from air on a refracting surface of a prism of angle 75° and refractive index $n_0 = \sqrt{3}$. The other refracting surface of a prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of $\theta \leq 60°$. The value of n^2 is_______.



Ans. 1.50

Sol. At θ = 60° ray incidents at critical angle at second surface



So, $\sin\theta = \sqrt{3}\,\sin r_1$

$$\frac{\sqrt{3}}{2} = \sqrt{3}\sin r_1$$

r_1 = 30°

r_2 = 45° = C

$\sqrt{3}$ sin 45° = n sin 90°

$$n = \sqrt{\frac{3}{2}} \Rightarrow n^2 = \frac{3}{2}$$

11. Suppose a ${}^{226}_{88}Ra$ nucleus at rest and in ground state undergoes α- decay to a ${}^{222}_{86}Rn$ nucleus in its excited state. The kinetic energy of the emitted α particle is found to be 4.44 MeV. ${}^{222}_{86}Rn$ nucleus then goes to its ground state by γ- decay. The energy of the emitted γ photon is _______ keV, [Given: atomic mass of ${}^{226}_{88}Ra$ = 226.005u , atomic mass of ${}^{222}_{86}Rn$ = 222.000 u, atomic mass of α particle = 4.000 u, 1 u = 931 MeV/c^2, c is speed of the light]

Ans. 135.00 (130 to 140)

Sol. $Ra^{226} \to Rn^{222} + \alpha$

Q = (226.005 – 222 – 4) 931 MeV

= 4.655 MeV

$$K_\alpha = \frac{A-4}{A}(Q - E_\gamma)$$

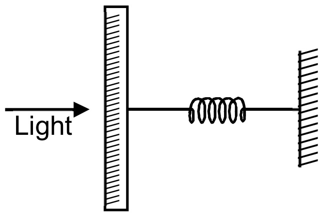
$$4.44\text{ MeV} = \frac{222}{226}(Q - E_\gamma)$$

$$Q - E_\gamma = (4.44)\left(\frac{226}{222}\right)\text{MeV}$$

E_γ = 4.655 – 4.520

= 0.135 MeV = 135 KeV

12. A perfectly reflecting mirror of mass M mounted on a spring constitutes a spring-mass system of angular frequency Ω such that $\frac{4\pi M\Omega}{h} = 10^{24}\,m^{-2}$ with h as Planck's constant. N photons of wavelength $\lambda = 8\pi \times 10^{-5}$ m strike the mirror simultaneously at normal incidence such that the mirror gets displaced by 1μm. If the value of N is x × 10^{12}, then the value of x is________. [Consider the spring as massless]



Ans. 1.00

Sol. Let momentum of one photon is p and after reflection velocity of the mirror is v.

Conservation of linear momentum

$N p \hat{i} = -N p \hat{i} + m v \hat{i}$

$m v \hat{i} = 2 p N \hat{i}$

mv = 2Np .....(1)

Since v is velocity of mirror (spring mass system) at mean position,

$v = A\Omega$

Where A is maximum deflection of mirror from mean position. (A = 1μm) and Ω is angular frequency of mirror spring system,

Momentum of 1 photon, $p = \frac{h}{\lambda}$

mv = 2Np ....(i)

$mA\Omega = 2N\frac{h}{\lambda}$

$N = \frac{m\Omega}{h} \times \frac{\lambda A}{2}$

Given, $\frac{m\Omega}{h} = \frac{10^{24}}{4\pi} m^{-2}$

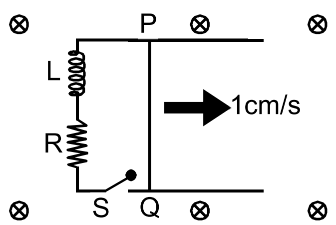
$\lambda = 8\pi \times 10^{-6}$ m

$N = \frac{10^{24}}{4\pi} \times \frac{8\pi \times 10^{-6} \times 10^{-6}}{2}$

$N = 10^{12} = x \times 10^{12}$

13. A 10 cm long perfectly conducting wire PQ is moving, with a velocity 1cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor L = 1 mH and a resistance R=1Ω as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field B = 1 T perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is $x \times 10^{-3}$ A, where the value of x is_______.

[Assume the velocity of wire PQ remains constant (1 cm/s) after key S is closed. Given : $e^{-1} = 0.37$, where e is base of the natural logarithm]



Ans. 0.63 (0.62 to 0.64)

Sol. Since velocity of PQ is constant. So emf developed across it remains constant.

ε = Blv where l = length of wire PQ

Current at any time t is given by

$$i = \frac{\varepsilon}{R}\left(1-e^{-Rt/L}\right)$$

$$i = \frac{Blv}{r}\left(1-e^{Rt/L}\right)$$

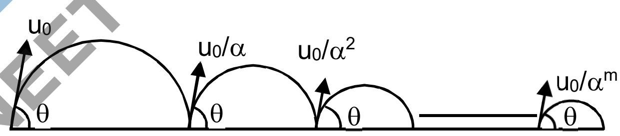
$$= 1\times\left(\frac{10}{100}\right)\times\left(\frac{1}{100}\right)\times\frac{1}{1}\left(1-e^{\frac{-1\times10^{-3}}{1\times10^{-3}}}\right)$$

$$= \frac{1}{100}\times\left(1-e^{-1}\right)$$

$$= \frac{1}{100}\times(1-0.37)$$

$i = 0.63 \times 10^{-3}$ A $\Rightarrow$ x = 0.63

14. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, ball rebounds at the same angle θ but with a reduced speed of u_0-α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is 0.8 V_1, the value of a is______



Ans. 4.00

Sol. Average velocity = $\dfrac{\text{Total displacement}}{\text{Total time}}$

Total time taken = $t_1 + t_2 + t_3 + ...$

$$= t_1 + \frac{t_1}{\alpha} + \frac{t_1}{\alpha^2} + ...$$

Total time = $\dfrac{t_1}{1-\dfrac{1}{\alpha}}$

Total displacement = $v_1t_1 + v_2t_2 + ...$

$= v_1t_1 + \dfrac{v_1}{\alpha}.\dfrac{t_1}{\alpha} + ...$

$= \dfrac{v_1t_1}{1-\dfrac{1}{\alpha^2}}$

On solving

$\langle v \rangle = \dfrac{v_1\alpha}{\alpha+1} = 0.8\, v_1$

$\alpha = 4.00$

SECTION–3 : (Maximum Marks : 12)

- This section contains TWO (02) List-Match sets.
- Each List-Match set has Two (02) Multiple Choice Questions.
- Each List-Match set has two lists : List-I and List-II
- List-I has Four entries (I), (II), (III) and (IV) and List-II has Six entries (P), (Q), (R), (S), (T) and (U)
- **FOUR** options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question wdill be evaluated according to the following marking scheme

Full Marks	:+ 3	If ONLY the option corresponding to the correct combination is chosen;
Zero Marks	:0	If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:–1	In all other cases.

15. Answer the following by appropriately matching the lists based on the information given in the paragraph

A musical instrument is made using four different metal strings, 1,2,3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

	List-I		**List-II**
(I)	String-1 (μ)	(P)	1
(II)	String-2 (2μ)	(Q)	1/2
(III)	String-3 (3μ)	(R)	$1/\sqrt{2}$

(IV)	String-4 (4μ)	(S)	$1/\sqrt{3}$
		(T)	3/16
		(U)	1/16

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be,

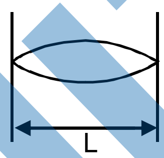
(1*) I $\rightarrow$ P, II $\rightarrow$ R, III $\rightarrow$ S, IV $\rightarrow$ Q

(2) I $\rightarrow$ Q, II $\rightarrow$ S, III $\rightarrow$ R, IV $\rightarrow$ P

(3) I $\rightarrow$ P, II $\rightarrow$ Q, III $\rightarrow$ T, IV $\rightarrow$ S

(4) I $\rightarrow$ Q, II $\rightarrow$ P, III $\rightarrow$ R, IV $\rightarrow$ T

Sol. For fundamental mode



$$\frac{\lambda}{2} = L \Rightarrow \lambda = 2L$$

$$f = \frac{V}{\lambda} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

For string (1)

$$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \Rightarrow (P)$$

From string (2)

$$f = \frac{1}{2L}\sqrt{\frac{T}{2\mu}} = \frac{f_0}{\sqrt{2}} \Rightarrow (R)$$

For string (3)

$$f = \frac{1}{2L}\sqrt{\frac{T}{3\mu}} = \frac{f_0}{\sqrt{3}} \Rightarrow (S)$$

Form string (4)

$$f = \frac{1}{2L}\sqrt{\frac{T}{4\mu}} = \frac{f_0}{2} \Rightarrow (Q)$$

16. Answer the following by appropriately matching the lists based on the information given in the paragraph

A musical instrument is made using four different metal strings, 1,2,3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

	List-I		**List-II**
(I)	String-1 (μ)	(P)	1
(II)	String-2 (2μ)	(Q)	1/2

(III)	String-3 (3μ)	(R)	$1/\sqrt{2}$
(IV)	String-4 (4μ)	(S)	$1/\sqrt{3}$
		(T)	3/16
		(U)	1/16

The length of the string 1, 2, 3 and 4 are kept fixed at L_0, $\frac{3L_0}{2}$, $\frac{5L_0}{4}$ and $\frac{7L_0}{4}$, respectively. Strings 1, 2, 3 and 4 are vibrated at their 1st, 3rd, 5th and 14th harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be

(1) I $\rightarrow$ T, II $\rightarrow$ Q, III $\rightarrow$ R, IV $\rightarrow$ U

(2) I $\rightarrow$ P, II $\rightarrow$ Q, III $\rightarrow$ R, IV $\rightarrow$ T

(3) I $\rightarrow$ P, II $\rightarrow$ R, III $\rightarrow$ T, IV $\rightarrow$ U

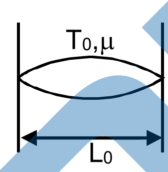
(4*) I $\rightarrow$ P, II $\rightarrow$ Q, III $\rightarrow$ T, IV $\rightarrow$ U

Sol. For string (1)

Length of string = L_0

It is vibrating in Ist harmonic i.e., fundamental mode.

$f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

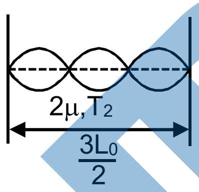


For string (2)

Length of string = $\frac{3L_0}{2}$

It is vibrating in IIIrd harmonic but frequency is still f_0

$f_0 = \frac{3v}{2L}$



$f_0 = \frac{3}{2\left(\frac{3L_0}{2}\right)}\sqrt{\frac{T_2}{2\mu}}$

$\Rightarrow f_0 = \frac{1}{L_0}\sqrt{\frac{T_2}{2\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

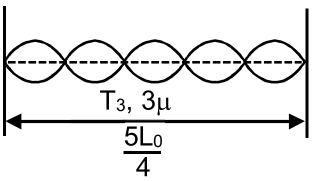
$\Rightarrow T_2 = \frac{T_0}{2}$

For string (3)

Length of string = $\frac{5L_0}{4}$

If is vibrating in 5th harmonic but frequency is still f_0

$f_0 = \frac{5V}{2L}$



$$\Rightarrow f_0 = \frac{5}{2\left(\frac{5L_0}{4}\right)}\sqrt{\frac{T_3}{3\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}\,]$$

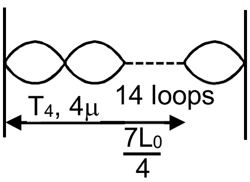
$$\Rightarrow \frac{2}{L_0}\sqrt{\frac{T_3}{3\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$T_3 = \frac{3T_0}{16}$$

For string (4)

Length of string = $\frac{7L_0}{4}$

It is vibrating in 14th harmonic but frequency is still f_0



$$f_0 = \frac{14v}{2L}$$

$$\Rightarrow f_0 = \frac{14}{2\left(\frac{7L_0}{4}\right)}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$\Rightarrow \frac{4}{L_0}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$\Rightarrow T_4 = \frac{T_0}{16}$$

17. Answer the following by appropriately matching the lists based on the information given in the paragraph.

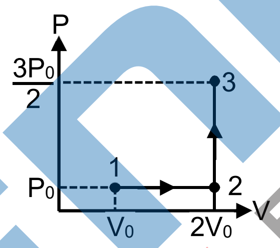
In a thermodynamics process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R\ln\left(\frac{T}{T_A}\right) + R\ln\left(\frac{V}{V_A}\right)$.

Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

	List-I		List-II
(I)	Work done by the system in process $1 \to 2 \to 3$	(P)	$\frac{1}{3}RT_0 \ln 2$
(II)	Change in internal energy in process $1 \to 2 \to 3$	(Q)	$\frac{1}{3}RT_0$
(III)	Heat absorbed by the system in process $1 \to 2 \to 3$	(R)	RT_0
(IV)	Heat absorbed by the system in process $1 \to 2$	(S)	$\frac{4}{3}RT_0$
		(T)	$\frac{1}{3}RT_0(3+\ell n2)$
		(U)	$\frac{5}{6}RT_0$

If the process carried out on one mole of monatomic ideal gas is as shown in figure in the PV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,



(1) $I \to Q, II \to R, III \to P, IV \to U$ (2) $I \to S, II \to R, III \to Q, IV \to T$

(3*) $I \to Q, II \to R, III \to S, IV \to U$ (4) $I \to Q, II \to S, III \to R, IV \to U$

Sol. (I) Degree of freedom f = 3

Work done in any process = Area under P-V graph

$\Rightarrow$ Work done in $1 \to 2 \to 3 = P_0V_0 = \frac{RT_0}{3}$

(II) Change in internal energy $1 \to 2 \to 3$

$\Delta U = nC_v\Delta T = \frac{f}{2}nR\Delta T = \frac{f}{2}(P_fV_f - P_iV_i) = \frac{3}{2}\left(\frac{3P_0}{2}2V_0 - P_0V_0\right) = 3P_0V_0$

$\Delta U = RT_0$

(III) Heat absorbed in $1 \to 2 \to 3$

For any process, I^{st} law of thermodynamics

$\Delta Q = \Delta W + \omega$

$\Delta Q = RT_0 + \frac{RT_0}{3}$

$\Delta Q = \frac{4RT_0}{3}$

(IV) Heat absorbed in process $1 \to 2$

$\Delta Q = \Delta U + W = \frac{f}{2}(P_fV_f - P_iV_i) + W$

$= \frac{3}{2}(P_0 2V_0 - P_0V_0) + P_0V_0 = \frac{5}{2}P_0V_0 = \frac{5}{2}\left(\frac{RT_0}{3}\right)$

$\Delta Q = \frac{5RT_0}{6}$

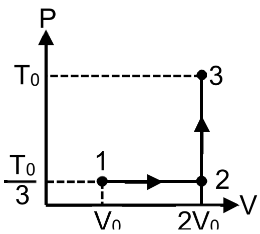
18. Answer the following by appropriately matching the lists based on the information given in the paragraph.

In a thermodynamics process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R\ell n\left(\frac{T}{T_A}\right) + R\ell n\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

List-I		List-II	
I)	Work done by the system in process $1 \to 2 \to 3$	(P)	$\frac{1}{3}RT_0 \ln 2$
(II)	Change in internal energy in process $1 \to 2 \to 3$	(Q)	$\frac{1}{3}RT_0$
(III)	Heat absorbed by the system in process $1 \to 2 \to 3$	(R)	RT_0
(IV)	Heat absorbed by the system in process $1 \to 2$	(S)	$\frac{4}{3}RT_0$
		(T)	$\frac{1}{3}RT_0(3 + \ell n2)$
		(U)	$\frac{5}{6}RT_0$

If the process carried out on one mole of monatomic ideal gas is as shown in the TV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,



(1*) I → P, II → R, III → T, IV → T

(2) I → P, II → R, III → T, IV → S

(3) I → S, II → T, III → Q, IV → U

(4) I → P, II → T, III → Q, IV → T

Sol. Process 1 → 2 is isothermal (temperature constant)

Process 2 → 3 is isochoric (volume constant)

(I) Work done in 1 → 2 → 3

$W = W_{1\to2} + W_{2\to3}$

$= nRT\,\ell n\left(\frac{V_f}{V_i}\right) + W_{2\to3}$

$= \frac{RT_0}{3}\ell n\left(\frac{2V_0}{V_0}\right) + 0$

$W = \frac{RT_0}{3}\ \ell n2$

(II) ΔU in 1 → 2 → 3

$\Delta U = \frac{f}{2}\ nR\,(T_f - T_i)$

$= \frac{3}{2}R\left(T_0 - \frac{T_0}{3}\right) = \frac{3}{2}R\left(\frac{2T_0}{3}\right)$

$\Delta U = RT_0$

(III) For any system, first law of thermodynamics

for 1 → 2 → 3

$\Delta Q = \Delta U + W$

$\Delta Q = RT_0 + \frac{RT_0}{3}\ell n2$

$\Delta Q = \frac{RT_0}{3}\ (3 + \ln 2)$

(IV) For process 1 → 2 (isothermal)

$\Delta Q = \Delta U + W$

$= \frac{f}{2}\ nR\,(T_f - T_i) + nRT\ \ln\,(V_f / V_i) = 0 + R\left(\frac{T_0}{3}\right)\ell n\left(\frac{2v_0}{v_0}\right)$

$\Delta Q = \frac{RT_0}{3}\ell n2$

PART-B: CHEMISTRY

SECTION-1:(Maximum Marks: 32)

- This section contains EIGHT (08) questions.
- Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s)
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:+ 4	If only (all) the correct option(s) is (are) chosen.
Partial Marks	:+3	If all the four options are correct but ONLY three options are chosen.
Partial Marks	:+2	If three or more options are correct but ONLY two options are chosen and both of which are correct.
Partial Marks	:+1	If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	:0	If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:–1	In all other cases.

For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;

choosing ONLY (A) and (B) will get +2 marks;

choosing ONLY (A) and (D) will get +2 marks;

choosing ONLY (B) and (D) will get +2 marks;

choosing ONLY (A) will get +1 marks;

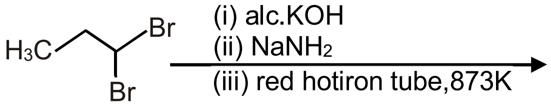
choosing ONLY (B) will get +1 marks;

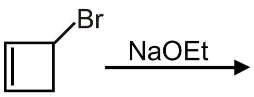
choosing ONLY (D) will get +1 marks;

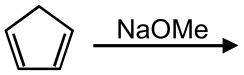
choosing no option (i.e. the question is unanswered) will get 0 marks, and

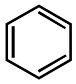
choosing any other combination of options will get –1 mark.

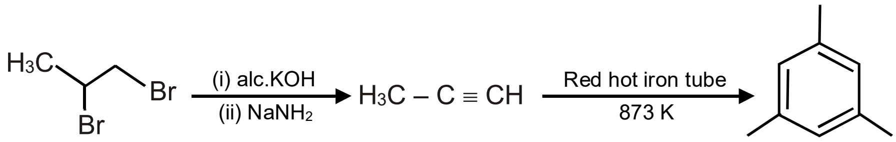
19. Choose the correct option(s) that give(s) an aromatic compound as the major product.

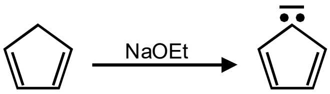
(A*) 

(B) 

(C*) 

(D)  + Cl_2 (excess) $\xrightarrow{\text{UV,500K}}$

Sol. (A) 

(C) 

20. Choose the correct option(s) from the following

(A*) Teflon is prepared by heating tetrafluoroethene in presence of a persulphate catalyst at high pressure

(B) Natural rubber is polyisoprene containing trans alkene units

(C) Cellulose has only α - D-glucose units that are joined by glycosidic linkages

(D*) Nylon-6 has amide linkages

21. Consider the following reactions (unbalanced)

Zn + hot conc. $H_2SO_4 \rightarrow$ G + R + X

Zn + conc. NaOH $\rightarrow$ T + Q

G + H_2S + $NH_4OH \rightarrow$ Z (a precipitate) + X + Y

Choose the correct option(s).

(A*) Bond order of Q is 1 in its ground state

(B*) Z is dirty white in colour

(C) The oxidation state of Zn in T is +1

(D*) R is a V-shaped molecule

Sol. $Zn + \underset{\text{(hot \& conc.)}}{H_2SO_4} \longrightarrow \underset{(G)}{ZnSO_4} + \underset{(R)}{SO_2} + \underset{(X)}{H_2O}$

$Zn + \text{conc. } NaOH \longrightarrow \underset{(T)}{Na_2ZnO_2} + \underset{(Q)}{H_2}$

$ZnSO_4 + H_2S + 2NH_4OH(aq.) \longrightarrow \underset{(Z)}{ZnS}\downarrow + \underset{(Y)}{(NH_4)_2SO_4} + \underset{(X)}{2H_2O}$

22. With reference to aqua regia, choose the correct option(s):

(1*) Aqua regia is prepared by mixing conc. HCl and conc. HNO_3 in 3 : 1 (v/v) ratio

(2*) The yellow colour of aqua regia is due to presence of NOCl and Cl_2

(3) Reaction of gold with aqua regia produces NO_2 in the absence of air

(4*) Reaction of gold with aqua regia produces an anion having Au in +3 oxidation state.

Sol. Au + HNO_3 + 3HCl $\longrightarrow$ $HAuCl_4$ + NO + H_2O

Yellow colour of aqua – regia is due to NOCl and Cl_2.

In aqua regia HCl and HNO_3 are in 3:1 molar ratio

23. The ground state energy of hydrogen atom is –13.6 eV. Consider an electronic state Ψ of He^+ whose energy, azimuthal quantum number and magnetic quantum number are –3.4 eV, 2 and 0, respectively. Which of the following statement(s) is(are) true for the state Ψ ?

(1) The nuclear charge experienced by the electron in this state is less than 2e, where e is the magnitude of the electronic charge.

(2) It has 3 radial nodes

(3*) It is a 4d state

(4*) It has 2 angular nodes

Sol. $-3.4 = \dfrac{-13.6 \times 2^2}{n^2}$

n = 4

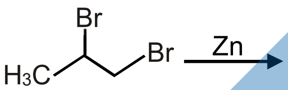
$\ell = 2$

Subshell = 4d

Angular nodes = ℓ = 2

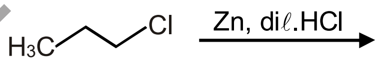
Radial nodes = $n - \ell - 1 = 4 - 2 - 1 = 1$

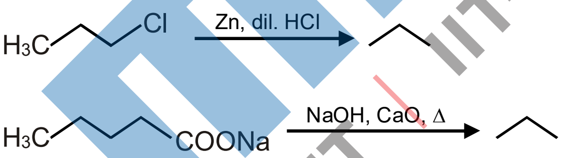
24. Which of the following reactions produce(s) propane as a major product?

(A) 

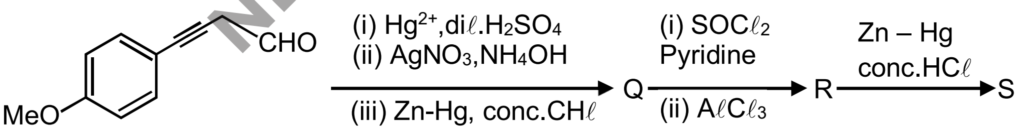
(B*) 

(C) 

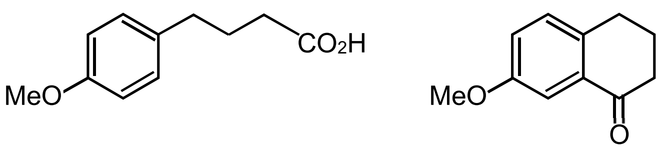
(D*) 

Sol. 

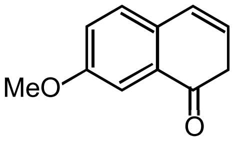
25. Choose the correct option(s) for the following reaction sequence

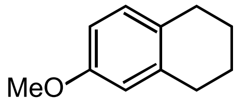


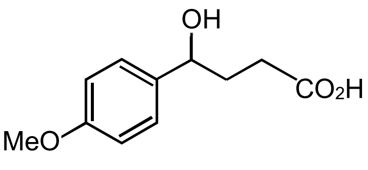
Consider Q, R and S as major products

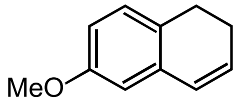
(A*) 

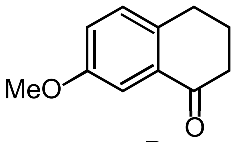
Q R

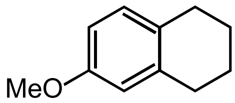
(B) 
R

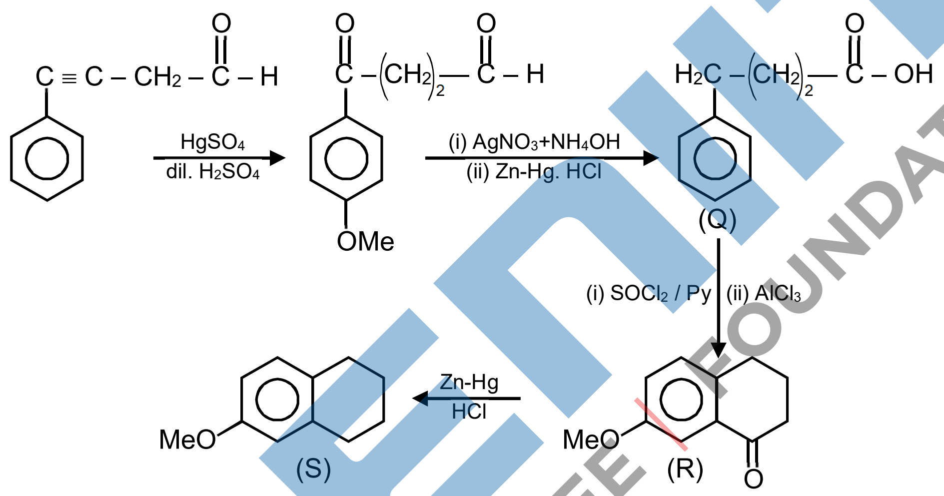

S

(C) 
Q


S

(D*) 
R


S

Sol. 

26. The cyanide process of gold extraction involves leaching out gold from its ore with CN^- in the presence of Q in water to form R. Subsequently, R is treated with T to obtain Au and Z. Choose the correct option(s).

(1*) Z is $[Zn(CN)_4]^{2-}$ (2) R is $[Au(CN)_4]^-$ (3*) Q is O_2 (4*) T is Zn

Sol. $Au \xrightarrow[H_2O]{CN^-, Q} R \xrightarrow{T} Au + Z$

$Au + 2CN^- + Q_2 \longrightarrow [Au(CN)_2]^-$

$[Au(CN)_2]^- + Zn \longrightarrow Au + [Zn(CN)_4]^{-2}$

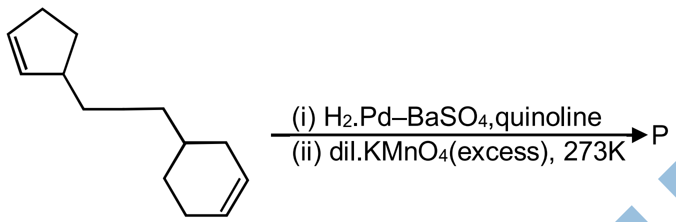
SECTION-2 : (Maximum Marks: 18)

- This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round-off the value to Two decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks :+ 3 If ONLY the correct numerical value is entered

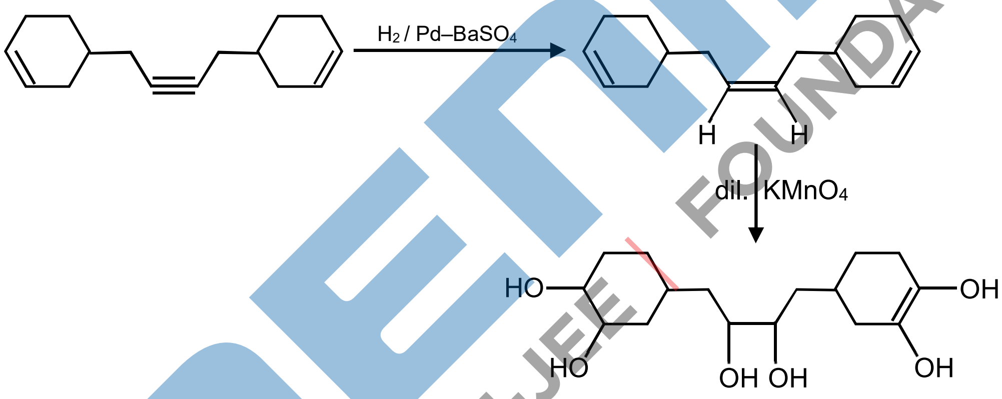
Zero Marks :0 In all other cases

27. Total number of hydroxyl groups present in a molecule of the major product P is ___



Ans. 6.00

Sol.



28. The amount of water produced (in g) in the oxidation of 1 mole of rhombic sulphur by conc.HNO_3 to a compound with the highest oxidation state of sulphur is ___

(Given data : Molar mass of water = 18 g mol^{-1})

Ans. 288

Sol. $S_8 + 48HNO_3 \longrightarrow 8H_2SO_4 + 48NO_2 + 16H_2O$

Mass of H_2O = 18 × 16 = 288 gram

29. The decomposition reaction $2N_2O_5(g) \xrightarrow{\Delta} 2N_2O_4(g) + O_2(g)$ is started in a closed cylinder under isothermal isochoric condition at an initial pressure of 1 atm. After Y × 10^3 s, the pressure inside the cylinder is found to be 1.45 atm. If the rate constant of the reaction is 5 × 10^{-4} s^{-1}, assuming ideal gas behavior, the value of Y is ___

Ans. 2.30

Sol. Unit of K represent it is first order reaction.

$$2N_2O_5 \longrightarrow 2N_2O_4 + O_2$$

t = 0	1	0	0
t = t	1 – P	P	P / 2

$$1-P+P+\frac{P}{2}=1.45$$

$$\frac{P}{2}=0.45, p=0.9$$

$$t=\frac{2.303}{2\times5\times10^{-4}}\log\frac{1}{1-P}$$

$$y\times10^{-3}=\frac{2.303}{2\times5\times10^{-4}}\log\frac{1}{1-0.9}=\frac{2.303}{2\times5\times10^{-4}}\log10$$

Y = 2.30

30. The mole fraction of urea in an aqueous urea solution containing 900 g of water is 0.05. If the density of the solution is 1.2 g cm^{-3}, the molarity of urea solution is ___

(Given data : Molar masses of urea and water are 60 g mol^{-1} and 18 g mol^{-1}, respectively)

Ans. 2.98

Sol. Let mole of urea = x

$$\frac{x}{x+\frac{900}{18}}=0.05, x=\frac{50}{19}$$

$$\text{Mass of solution }=\frac{50}{19}\times60+900=\frac{20100}{19}$$

$$\text{Volume of solution }=\frac{20100}{19\times1.2}\text{ml}$$

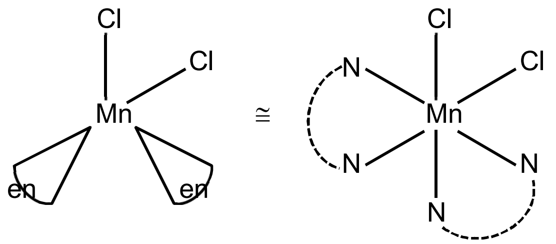
$$=\frac{20.1}{19\times1.2}\text{litre}$$

$$\text{Molarity }=\frac{\frac{50}{19}}{\frac{20.1}{19\times1.2}}=\frac{50\times1.2}{20.1}=\frac{60}{20.1}\cong2.985$$

31. Total number of cis N–Mn–Cl bond angles (that is, Mn–N and Mn–Cl bonds in cis positions) present in a molecule of cis-$[Mn(en)_2Cl_2]$ complex is ____ (en = $NH_2CH_2CH_2NH_2$)

Ans. 6.00

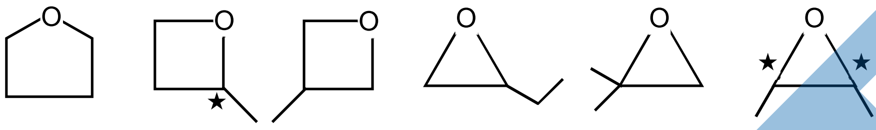
Sol.



32. Total number of isomers, considering both structural and stereoisomers, of cyclic ethers with the molecular formula C_4H_8O is ___

Ans. 10.00

Sol.



(1) (2) (1) (2) (1) (3)

SECTION–3 : (Maximum Marks : 12)

- This section contains TWO (02) List-Match sets.
- Each List-Match set has Two (02) Multiple Choice Questions.
- Each List-Match set has two lists : List-I and List-II
- List-I has Four entries (I), (II), (III) and (IV) and List-II has Six entries (P), (Q), (R), (S), (T) and (U)
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme

Full Marks	:	+ 3 If ONLY the option corresponding to the correct combination is chosen;
Zero Marks	:	0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:	–1 In all other cases.

33. Answer the following by appropriately matching the lists based on the information given in the paragraph.

Consider the Bohr's model of a one-electron atom where the electron moves around the nucleus. In the following List-I contains some quantities for the n^{th} orbit of the atom and List-II contains options showing how they depend on n.

	List - I	List-II	
(I)	Radius of the n^{th} orbit	(P)	$\propto n^{-2}$
(II)	Angular momentum of the electron in the n^{th} orbit	(Q)	$\propto n^{-1}$
(III)	Kinetic energy of the electron in the n^{th} orbit	(R)	$\propto n^{0}$
(IV)	Potential energy of the electron in the n^{th} orbit	(S)	$\propto n^{1}$

(T) $\propto n^2$

(U) $\propto n^{1/2}$

Which of the following options has the correct combination considering List-I and List-II ?

(1*) (III), (P) (2) (IV), (Q) (3) (III) , (S) (4) (IV), (U)

34. Answer the following by appropriately matching the lists based on the information given in the paragraph.

Consider the Bohr's model of a one-electron atom where the electron moves around the nucleus. In the following List-I contains some quantities for the n^{th} orbit of the atom and List-II contains options showing how they depend on n.

List - I		**List-II**	
(I)	Radius of the n^{th} orbit	(P)	$\propto n^{-2}$
(II)	Angular momentum of the electron in the n^{th} orbit	(Q)	$\propto n^{-1}$
(III)	Kinetic energy of the electron in the n^{th} orbit	(R)	$\propto n^0$
(IV)	Potential energy of the electron in the n^{th} orbit	(S)	$\propto n^1$
		(T)	$\propto n^2$
		(U)	$\propto n^{1/2}$

Which of the following options has the correct combination considering List-I and List-II ?

(1*) (I), (T) (2) (II), (R) (3) (II) , (Q) (4) (I), (P)

Sol. **(33-34)**

$$KE = \frac{13.6Z^2}{n^2} eV / atom$$

$$PE = \frac{-2 \times 13.6Z^2}{n^2} eV / atom$$

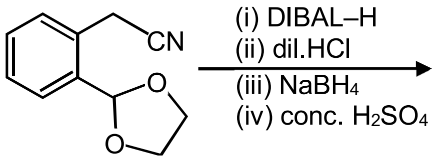
$$Radius = 0.529 \frac{n^2}{Z} Å$$

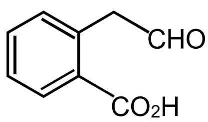
$$\text{Angular momentum of electron (mvr)} = \frac{nh}{2\pi}$$

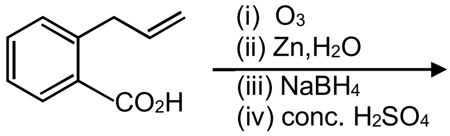
35. Answer the following by appropriately matching the lists based on the information given in the paragraph.

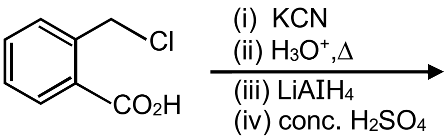
List-I includes starting materials and reagents of selected chemical reactions. List-II gives structures of compounds that may be formed as intermediate products and / or final products from the reactions of list-I.

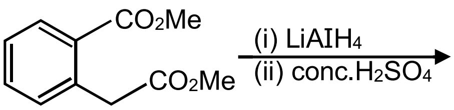
List-I **List-II**

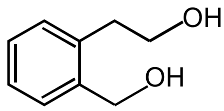
(I) 

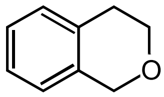
(P) 

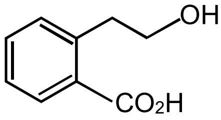
(II) 

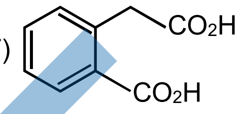
(III) 

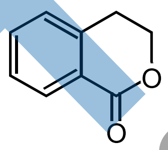
(IV) 

(Q) 

(R) 

(S) 

(T) 

(U) 

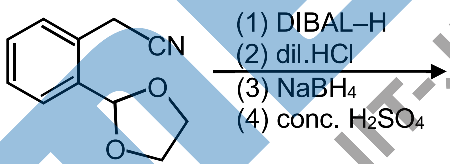
Which of the following options has correct combination considering List-I and List-II?

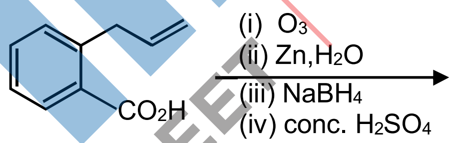
(1*) (IV), (Q), (R) (2) (III), (S), (R) (3) (III), (T), (U) (4) (IV), (Q), (U)

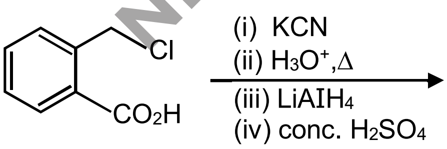
36. Answer the following by appropriately matching the lists based on the information given in the paragraph.

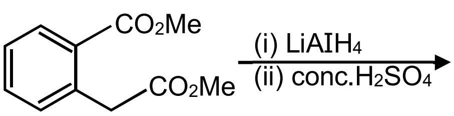
List-I includes starting materials and reagents of selected chemical reactions. List-II gives structures of compounds that may be formed as intermediate products and / or final products from the reactions of list-I.

List-I

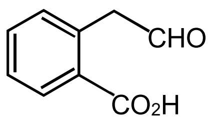
(I) 

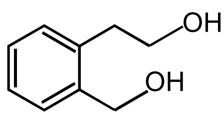
(II) 

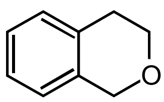
(III) 

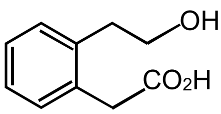
(IV) 

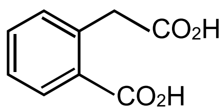
List-II

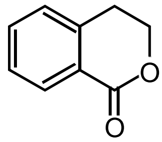
(P) 

(Q) 

(R) 

(S) 

(T) 

(U) 

Which of the following options has correct combination considering List-I and List-II?

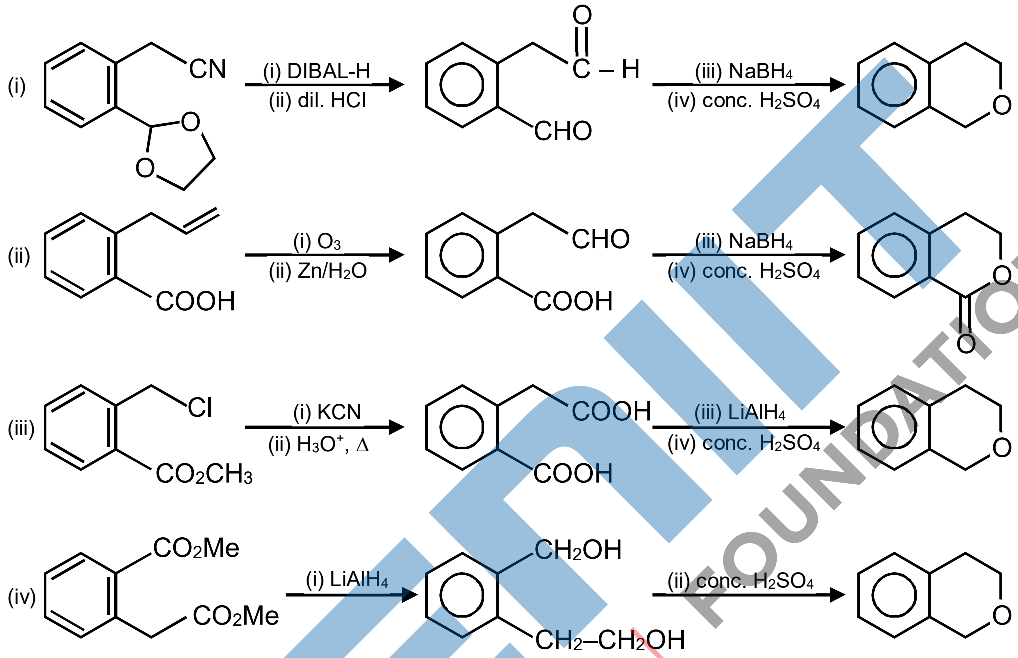
(1) (I), (Q), (T), (U)

(2) (II), (P), (S), (T)

(3) (I), (S), (Q), (R)

(4*) (II), (P), (S), (U)

Sol. **(35-36)**



PART-C: MATHEMATICS

SECTION-1 : (Maximum Marks: 32)

– This section contains EIGHT (08) questions.

– Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).

– For each question, choose the option(s) corresponding to (all) the correct answer(s).

– Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is(are) chosen.

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : –1 In all other cases.

– For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;

choosing ONLY (A) and (B) will get +2 marks;

choosing ONLY (A) and (D) will get +2 marks;

choosing ONLY (B) and (D) will get +2 marks;

choosing ONLY (A) will get +1 marks;

choosing ONLY (B) will get +1 marks;

choosing ONLY (D) will get +1 marks;

choosing no option (i.e. the question is unanswered) will get 0 marks, and

choosing any other combination of options will get –1 mark.

37. Let $f : R \to R$ be a function. We say that f has

PROPERTY 1 if $\lim_{h\to 0} \frac{f(h)-f(0)}{\sqrt{|h|}}$ exists and is finite, and

PROPERTY 2 if $\lim_{h\to 0} \frac{f(h)-f(0)}{h^2}$ exists and is finite.

Then which of the following options is/are correct?

(1) $f(x) = \sin x$ has PROPERTY 2

(2*) $f(x) = x^{2/3}$ has PROPERTY 1

(3) $f(x) = x|x|$ has PROPERTY 2

(4*) $f(x) = |x|$ has PROPERTY 1

Ans. 2, 4

Sol. Property 1: $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{\sqrt{|h|}}$ = exist and finite

(B) f(x) = $x^{2/3}$, $\lim\limits_{h\to 0}\dfrac{h^{\frac{2}{3}}-0}{\sqrt{|h|}}=\lim\limits_{h\to 0}\dfrac{|h|^{2/3}}{\sqrt{|h|}}=0$

(D) f(x) = | x |, $\lim\limits_{h\to 0}\dfrac{|h|-0}{\sqrt{|h|}}\Rightarrow \lim\limits_{h\to 0}\sqrt{|h|}=0$

Property 2: $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h^2}$ = exists and finite.

(A) f(x) = x | x |, $\lim\limits_{h\to 0}\dfrac{h|h|-0}{h^2}=\left[\begin{array}{l}\text{RHL}=\lim\limits_{h\to 0}\dfrac{h^2}{h^2}=1\\ \text{LHL}=\lim\limits_{h\to 0}\dfrac{-h^2}{h^2}=-1\end{array}\right.$

(C) f(x) = sin x, $\lim\limits_{h\to 0}\dfrac{\sin h-0}{h^2}$ = DNE.

38. For non-negative integers n, let

$$f(n)=\frac{\sum\limits_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum\limits_{k=0}^{n}\sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in [0, π], which of the following options is/are correct?

(1*) f (4) = $\dfrac{\sqrt{3}}{2}$

(2*) If α = tan ($\cos^{-1}$ f(6)), then $\alpha^2+2\alpha-1=0$

(3*) sin (7$\cos^{-1}$ f(5)) = 0

(4) $\lim\limits_{n\to\infty}f(n)=\dfrac{1}{2}$

Ans. 1, 2, 3

Sol. $$f(n)=\frac{\sum\limits_{k=0}^{n}\left(\cos\left(\frac{\pi}{n+2}\right)-\cos\left(\frac{2k+3}{n+2}\right)\pi\right)}{\sum\limits_{k=0}^{n}\left(1-\cos\left(\frac{2k+2}{n+2}\right)\pi\right)}$$

$$f(n)=\frac{(n+2)\cos\left(\frac{\pi}{n+2}\right)-\left(\sum\limits_{k=0}^{n}\cos\left(\frac{2k+3}{n+2}\right)\pi\right)}{(n+1)-\left(\sum\limits_{k=0}^{n}\cos\left(\frac{2k+2}{n+2}\right)\pi\right)}$$

$$f(n)=\frac{(n+1)\cos\left(\frac{\pi}{n+2}\right)-\left(\frac{\sin\left(\frac{(n+1)\pi}{n+2}\right)}{\sin\left(\frac{\pi}{n+2}\right)}\cdot\cos\left(\frac{n+3}{n+2}\right)\pi\right)}{(n+1)-\left(\frac{\sin\left(\frac{(n+1)\pi}{n+2}\right)}{\sin\left(\frac{\pi}{n+2}\right)}\cdot\cos\left(\frac{2(n+2)\pi}{2(n+2)}\right)\right)}$$

$$f(n)=\frac{(n+1)\cos\left(\frac{\pi}{n+2}\right)+\cos\left(\frac{\pi}{n+2}\right)}{(n+1)+1}\Rightarrow g(x)=\cos\left(\frac{\pi}{n+2}\right)$$

(A) $\sin\left(7\cos^{-1}\left(\cos\frac{\pi}{7}\right)\right)=\sin\pi=0$

(B) $f(4)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

(C) $\lim\limits_{n\to\infty}\cos\left(\frac{\pi}{n+2}\right)=1$ = 1

(D) $\alpha=\tan\left(\cos^{-1}\cos\frac{\pi}{8}\right)=\sqrt{2}-1\Rightarrow\alpha+1=\sqrt{2}$

$\Rightarrow \alpha^2 + 2\alpha + 1 = 0$.

39. Let $x\in R$ and let $P=\begin{bmatrix}1&1&1\\0&2&2\\0&0&3\end{bmatrix}, Q=\begin{bmatrix}2&x&x\\0&4&0\\x&x&6\end{bmatrix}$ and $R = PQP^{-1}$.

Then which of the following options is/are correct?

(1) For x = 1, there exists a unit vector $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ for which $R\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$.

(2*) For x = 0, if $R\begin{bmatrix}1\\a\\b\end{bmatrix}=6\begin{bmatrix}1\\a\\b\end{bmatrix}$, then a + b = 5.

(3*) $\det R=\det\begin{bmatrix}2&x&x\\0&4&0\\x&x&5\end{bmatrix}+8$ for all $x\in R$

(4) There exists a real number x such that PQ = QP.

Ans. 2, 3

Sol. det (R) = det (PQP^{-1}) = (det P) (det Q) $\left(\frac{1}{\det P}\right)$ = det Q = $48 - 4x^2$

(1) For x = 1, det R = $44 \neq 0$

$\therefore$ for equation $R\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

We will have trivial solution

$\alpha = \beta = \gamma = 0$

(2) PQ = QP

$PQP^{-1} = Q$

R = Q

No value of x.

(3) $\det\begin{bmatrix}2 & x & x\\0 & 4 & 0\\x & x & 5\end{bmatrix} + 8 = (40 - 4x^2) + 8 = 48 - 4x^2 = \det R$, for all $x \in R$.

(4) $R = \begin{bmatrix}2 & 1 & \frac{2}{3}\\0 & 4 & \frac{4}{3}\\0 & 0 & 6\end{bmatrix}$

$(R - 6I)\begin{bmatrix}1\\a\\b\end{bmatrix} = O$

$\Rightarrow -4 + a + \frac{2b}{3} = 0 \Rightarrow -2a + \frac{4b}{3} = 0 \Rightarrow a = 2;\ b = 3$

$\therefore$ a + b = 5.

40. Let $f(x) = \frac{\sin \pi x}{x^2}$, $x > 0$

Let $x_1 < x_2 < x_3 < ... < x_n < ...$ be all the points of local maximum of f

and $y_1 < y_2 < y_3 < ... < y_n < ...$ be all the points of local minimum of f.

Then which of the following options is/are correct?

(1*) $x_n \in \left(2n, 2n + \frac{1}{2}\right)$ for every n

(2) $x_1 < y_1$

(3*) $x_{n+1} - x_n > 2$ for every n

(4*) $|x_n - y_n| > 1$ for every n

Ans. 1, 3, 4

Sol. $f(x) = \frac{\sin \pi x}{x^2}$

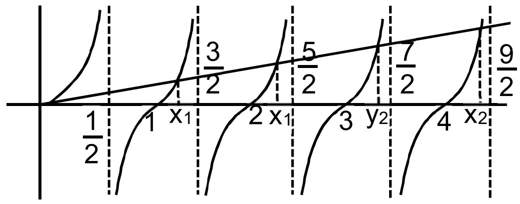
$f'(x) = \frac{2x\cos \pi x\left(\frac{\pi x}{2} - \tan \pi x\right)}{x^4}$

$\Rightarrow |x_n - y_n| > 1$ for every n

$x_1 > y_1$

$x_n \in \left(2n, 2n + \frac{1}{2}\right)$

$x_{n+1} - x_n > 2.$



41. Three lines

$L_1 : \vec{r} = \lambda\hat{i}, \lambda \in R,$
$L_2 : \vec{r} = \hat{k} + \mu\hat{j}, \mu \in R$ and
$L_3 : \vec{r} = \hat{i} + \hat{j} + v\hat{k}, v \in R$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear?

(1) $\hat{k} + \hat{j}$ (2) $\hat{k}$ (3*) $\hat{k} - \frac{1}{2}\hat{j}$ (4*) $\hat{k} + \frac{1}{2}\hat{j}$

Ans. 3, 4

Sol. Let P(λ, 0, 0); Q (0, μ, 1); R (1, 1, v) be points. L_1, L_2 and L_3 respectively

Since, P, Q, R are collinear, $\overrightarrow{PQ}$ is collinear with $\overrightarrow{OR}$

Hence, $\frac{-\lambda}{1} = \frac{\mu}{1-\mu} = \frac{1}{v-1}$

For every $\mu \in R - \{0, 1\}$, there exist unique $\lambda, v \in R$

Hence, Q cannot have coordinates (0, 1, 1) and (0, 0, 1).

42. Let f : R $\to$ R be given by f(x) = (x – 1) (x – 2) (x – 5). Define F(x) = $\int_0^x f(t)dt$, x > 0. Then which of the following options is/are correct?

(1*) F has a local minimum at x = 1

(2) F has two local maxima and one local minimum in $(0, \infty)$

(3*) $F(x) \neq 0$ for all $x \in (0, 5)$

(4*) F has a local maximum at x = 2

Ans. 1, 3, 4

Sol. f(x) = (x – 1) (x – 2) (x – 5)

F(x) = $\int_0^x f(t)dt$, x > 0

F'(x) = f(x) = (x – 1) (x – 2) (x – 5), x > 0

Clearly, F(x) has local minimum at x = 1, 5

F(x) has local maximum at x = 2

$f(x) = x^3 - 8x^2 + 17x - 10$

$\Rightarrow F(x) = \int_0^x \left(t^3 - 8t^2 + 17t - 10\right)dt$

$F(x) = \frac{x^4}{4} - \frac{8x^3}{3} + \frac{17x^2}{2} - 10x$

From the graph of y = F(x), clearly $F(x) \neq 0 \ \forall \ x \in (0, 5)$.

43. For $a \in R$, $|a| > 1$, let $\lim_{n\to\infty}\left(\frac{1+\sqrt[3]{2}+........+\sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(an+1)^2}+\frac{1}{(an+2)^2}+.......+\frac{1}{(an+n)^2}\right)}\right) = 54$. Then the possible value(s) of a is/are

(1*) 8 (2) 7 (3*) –9 (4) –6

Ans. 1, 3

Sol. $\lim_{n\to\infty}\frac{n^{\frac{1}{3}}\left(\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{1/3}\right)}{n^{\frac{7}{3}}\left(\sum_{r=1}^{n}\frac{1}{(an+r)^2}\right)} = 54 \Rightarrow \lim_{n\to\infty}\left(\frac{\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{1/3}}{\frac{1}{n}\sum_{r=1}^{n}\frac{1}{\left(a+\frac{r}{n}\right)^2}}\right) = 54$

$\Rightarrow \frac{\int_0^1 x^{\frac{1}{3}}dx}{\int_0^1 \frac{dx}{(a+x)^2}} = 54 \Rightarrow \frac{\frac{3}{4}}{\frac{1}{a(a+1)}} = 54 \Rightarrow a(a+1) = 72$

$\Rightarrow a^2 + a - 72 = 0 \Rightarrow a = -9, 8.$

44. Let $P_1 = I = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}, P_2 = \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}, P_3 = \begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}, P_4 = \begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}, P_5 = \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}, P_6 = \begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$

and $X = \sum_{k=1}^{6} P_k \begin{bmatrix}2&1&3\\1&0&2\\3&2&1\end{bmatrix} P_k^T$

where P_k^T denotes the transpose of the matrix P_k. Then which of the following options is/are correct?

(1) X – 30I is an invertible matrix

(2*) The sum of diagonal entries of X is 18

(3*) X is a symmetric matrix

(4*) If $X\begin{bmatrix}1\\1\\1\end{bmatrix} = \alpha\begin{bmatrix}1\\1\\1\end{bmatrix}$, then $\alpha = 30$

Ans. 2, 3, 4

Sol. Let $Q = \begin{bmatrix}2&1&3\\1&0&2\\3&2&1\end{bmatrix}$

$$X = \sum_{k=1}^{6}\left(P_k Q P_K^T\right)$$

$$X^T = \sum_{k=1}^{6}\left(P_k Q P_K^T\right)^T = X$$

X is symmetric

Let $R = \begin{bmatrix}1\\1\\1\end{bmatrix}$

$$XR = \sum_{k=1}^{6} P_K Q P_K^T R \quad [\because P_K^T R = R]$$

$$= \sum_{K=1}^{6} P_K QR = \left(\sum_{K=1}^{6} P_K\right) QR$$

$$\sum_{K=1}^{6} P_K = \begin{bmatrix}2 & 2 & 2\\2 & 2 & 2\\2 & 2 & 2\end{bmatrix};\ QR = \begin{bmatrix}6\\3\\6\end{bmatrix}$$

$$\Rightarrow XR = \begin{bmatrix}2 & 2 & 2\\2 & 2 & 2\\2 & 2 & 2\end{bmatrix}\begin{bmatrix}6\\3\\6\end{bmatrix} = \begin{bmatrix}30\\30\\30\end{bmatrix} = 30R$$

$\Rightarrow \alpha = 30$

$$\text{Trace } X = \text{Trace}\left(\sum_{k=1}^{6} P_K Q P_K^T\right) = \sum_{k=1}^{6} \text{Trace}\left(P_K Q P_K^T\right) = 6\ (\text{Trace } Q) = 18$$

$$X\,X\begin{bmatrix}1\\1\\1\end{bmatrix} = 30\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\Rightarrow (X - 30I)\begin{bmatrix}1\\1\\1\end{bmatrix} = O \Rightarrow |X - 30I| = 0$$

$\Rightarrow X - 30I$ is non-invertible.

SECTION-2 : (Maximum Marks: 18)

- This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round-off the value to Two decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct numerical value is entered.

  Zero Marks : 0 In all other cases.

45. The value of $\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right)$ in the interval $\left[\frac{-\pi}{4},\frac{3\pi}{4}\right]$ equals

Ans. 0.00

Sol. $\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\frac{1}{\cos\left(\frac{7\pi}{12}+\frac{k\pi}{12}\right)\cos\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)}\right)$

$$= \sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\frac{\sin\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}-\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\right)}{\cos\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\cdot\cos\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)}\right)$$

$$= \sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\tan\left(\frac{7\pi}{12}+(k+1)\frac{\pi}{2}\right)-\tan\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\right)$$

$$= \sec^{-1}\left(\frac{1}{4}\left(\tan\left(\frac{11\pi}{2}+\frac{7\pi}{12}\right)-\tan\left(\frac{7\pi}{12}\right)\right)\right) = \sec^{-1}\left(\frac{1}{4}\left(-\cot\frac{7\pi}{12}-\tan\frac{7\pi}{12}\right)\right)$$

$$= \sec^{-1}\left(\frac{1}{4}\left(\frac{-1}{\sin\frac{7\pi}{12}\cos\frac{7\pi}{12}}\right)\right) = \sec^{-1}\left(\frac{-1}{2}\times\frac{1}{\sin\frac{7\pi}{6}}\right) = \sec^{-1}(1) = 0.$$

46. Let | X | denote the number of elements in a set X. Let S = {1,2,3,4,5,6} be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A, B) such that $1 \le |B| < |A|$, equals

Ans. 422.00

Sol. $P\left(\frac{B}{A}\right) = P(B)$

$\Rightarrow \frac{(A\cap B)}{n(A)} = \frac{n(B)}{n(S)}$ ....(1)

$\Rightarrow$ n(A) should have 2 or 3 as prime factors

$\Rightarrow$ n(A) can be 2, 3, 4 or 6 as n(A) > 1

n(A) = 2 does not satisfy the constraint (1).

For n(A) = 3. n(B) = 2 and $n(A \cap B) = 1$

$\Rightarrow$ Number of ordered pair = ${}^6C_4 \times \frac{4!}{2!} = 180$

For n(A) = 4. n(B) = 3 and $n(A \cap B) = 2$

$\Rightarrow$ Number of ordered pairs = ${}^6C_5 \times \frac{5!}{2!\cdot 2!} = 180$

For n(A) = 6. n(B) can be 1, 2, 3, 4, 5.

$\Rightarrow$ Number of ordered pairs = $2^6 - 2 = 62$

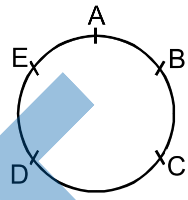
Total ordered pair = 180 + 180 + 62 = 422.

47. Five persons A,B,C,D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green ,then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is

Ans. 30.00

Sol. When 1R, 2B, 2G

$^5C_1 \times 2 = 10$



Other possibilities

1B, 2R, 2G

or 1G, 2R, 2B

So, total number of ways = 3 × 10 = 30.

48. Suppose det $\begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^nC_k\, k^2 \\ \sum_{k=0}^{n} {}^nC_k\, k & \sum_{k=0}^{n} {}^nC_k\, 3^k \end{bmatrix} = 0$, holds for some positive integer n.

Then $\sum_{k=0}^{n} \frac{{}^nC_k}{k+1}$ equals

Ans. 6.20

Sol. Suppose $\begin{vmatrix} \frac{n(n+1)}{2} & n(n-1)\cdot 2^{n-2} + n\cdot 2^{n-1} \\ n\cdot 2^{n-1} & 4^n \end{vmatrix} = 0$

$\frac{n(n+1)}{2} \cdot 4^n - n^2(n-1)\cdot 2^{2n-3} - n^2\, 2^{2n-2} = 0$

$\frac{n(n+1)}{2} - \frac{n^2(n-1)}{8} - \frac{n^2}{4} = 0$

$n^2 - 3n - 4 = 0 \Rightarrow n = 4$

Now, $\sum_{k=0}^{n} \frac{{}^nC_k}{k+1} = \sum_{k=0}^{4} \frac{k+1}{5} \cdot {}^5C_{k+1}\left(\frac{1}{k+1}\right) = \frac{1}{5}[{}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5]$

$= \frac{1}{5}[2^5 - 1] = \frac{31}{5} = 6.20.$

49. The value of the integral $\int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta} + \sqrt{\sin\theta}\right)^5} d\theta$ equals

Ans. 0.50

Sol. $I = \int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta} + \sqrt{\sin\theta}\right)^5} d\theta = \int_0^{\pi/2} \frac{3\sqrt{\sin\theta}}{\left(\sqrt{\cos\theta} + \sqrt{\sec\theta}\right)} d\theta$

$$2I = \int_0^{\pi/2} \frac{3d\theta}{\left(\sqrt{\cos\theta}+\sqrt{\sec\theta}\right)^4} = 3\int_0^{\pi/2} \frac{\sec^2\theta}{\left(1+\sqrt{\tan\theta}\right)^4}\, d\theta$$

Let 1 + $\sqrt{\tan\theta} = t$

$\frac{\sec^2\theta}{2\sqrt{\tan\theta}}$ dθ = dt $\Rightarrow$ $\sec^2\theta$ dθ = 2 (t – 1) dt

$$= 3\int_1^{\infty} \frac{2(t-1)}{t^4}\, dt = 6\int_1^{\infty}\left(t^{-3}-t^{-4}\right) dt$$

$$2I = 6\left(\frac{t^{-2}}{-2}-\frac{t^{-3}}{-3}\right)_1^{\infty} = 6\left[0-0-\left\{\frac{-1}{2}+\frac{1}{3}\right\}\right]$$

I = 0.50.

50. Let $\vec{a} = 2\hat{i}+\hat{j}-\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+\hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a}+\beta\vec{b}, \alpha,\beta \in R$. If the projection of $\vec{c}$ on the vector $\left(\vec{a}+\vec{b}\right)$ is $3\sqrt{2}$, then the minimum value of $\left(\vec{c}-(\vec{a}\times\vec{b})\right)\cdot\vec{c}$ equals

Ans. 18.00

Sol. $\vec{c} = (2\alpha+\beta)\hat{i}+\hat{j}(\alpha+2\beta)+\hat{k}(\beta-\alpha)$

$\frac{\vec{c}\cdot(\vec{a}+\vec{b})}{|\vec{a}+\vec{b}|} = 3\sqrt{2}$

$\Rightarrow \alpha + \beta = 2$ ....(1)

$\left(\vec{c}-(\vec{a}\times\vec{b})\right)\cdot\left(\alpha\vec{a}+\beta\vec{b}\right)$

$=|\vec{c}|^2 = \alpha^2|\vec{a}|^2 + \beta^2|b|^2 + 2\alpha\beta(\vec{a}\cdot\vec{b}) = 6(\alpha^2+\beta^2+\alpha\beta)$

$= 6(\alpha^2 + (2-\alpha)^2 + \alpha(2-\alpha)) = 6((\alpha-1)^2+3)$

$\Rightarrow$ Minimum value = 18.

SECTION–3 : (Maximum Marks : 12)

– This section contains TWO (02) List-Match sets.

– Each List-Match set has Two (02) Multiple Choice Questions.

– Each List-Match set has two lists : List-I and List-II

– List-I has Four entries (I), (II), (III) and (IV) and List-II has Six entries (P), (Q), (R), (S), (T) and (U)

– FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.

– Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 If ONLY the option corresponding to the correct combination is chosen.

Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);

Negative Marks : –1 In all other cases

51. Answer the following by appropriately matching the lists based on the information given in the paragraph

Let $f(x) = \sin(\pi\cos x)$ and $g(x) = \cos(2\sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order :

$$X = \{x : f(x) = 0\},\ Y = \{x : f'(x) = 0\}$$

$$Z = \{x : g(x) = 0\},\ W = \{x : g'(x) = 0\}.$$

List-I contains the sets X, Y, Z and W. List-II contains some information regarding these sets.

	List-I		List-II
(I)	X	(P)	$\supseteq\left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II)	Y	(Q)	an arithmetic progression
(III)	Z	(R)	NOT an arithmetic progression
(IV)	W	(S)	$\supseteq\left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
		(T)	$\supseteq\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
		(U)	$\supseteq\left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination ?

Options

(1) (I), (P), (R)　(2*) (II), (Q), (T)　(3) (II), (R), (S)　(4) (I), (Q), (U)

Ans. 2

52. Answer the following by appropriately matching the lists based on the information given in the paragraph

Let $f(x) = \sin(\pi\cos x)$ and $g(x) = \cos(2\sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order :

$$X = \{x : f(x) = 0\},\ Y = \{x : f'(x) = 0\}$$

$$Z = \{x : g(x) = 0\},\ W = \{x : g'(x) = 0\}.$$

List-I contains the sets X,Y,Z and W. List-II contains some information regarding these sets.

	List-I		List-II
(I)	X	(P)	$\supseteq\left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II)	Y	(Q)	an arithmetic progression
(III)	Z	(R)	NOT an arithmetic progression
(IV)	W	(S)	$\supseteq\left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
		(T)	$\supseteq\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$

(U) $\supseteq\left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination ?

Options

(1) (IV), (Q), (T) (2) (III), (P), (Q), (U) (3*) (IV), (P), (R), (S) (4) (III), (R), (U)

Ans. 3

Sol. f(x) = sin (π cos x)

X : {x : f(x) = 0}

f(x) = 0 $\Rightarrow$ sin (π cos x) = 0 $\Rightarrow$ cos x = n $\Rightarrow$ cos x = 1, – 1, 0 $\Rightarrow$ x = $\frac{n\pi}{2}$

$$X = \left\{\frac{n\pi}{2} : n \in N\right\} = \left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, ...\right\}$$

g(x) = cos (2π sin x)

Z = {x : g(x) = 0}

cos (2π sin x) = 0 $\Rightarrow$ 2π sin x = (2n + 1) $\frac{\pi}{2}$ $\Rightarrow \sin x = \frac{(2n+1)}{4}$

sin x = $\frac{-1}{4}, \frac{1}{4}, \frac{-3}{4}, \frac{3}{4}$

$$Z = \left\{n\pi \pm \sin^{-1}\left(\frac{1}{4}\right), n\pi \pm \sin^{-1}\left(\frac{3}{4}\right), n \in I\right\}$$

Y = {x : f '(x) = 0}

f(x) = sin (π cos x) $\Rightarrow$ f '(x) = cos (π cos x) · (– π sin x) = 0

sin x = 0 $\Rightarrow$ x = nπ.

cos (π cos x) = 0 $\Rightarrow$ πcos x = (2n + 1) $\frac{\pi}{2}$ $\Rightarrow \cos x = \frac{(2n+1)}{2} \Rightarrow \cos x = \frac{-1}{2}, \frac{1}{2}$

$$Y = \left\{n\pi, n\pi \pm \frac{\pi}{3}\right\} = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi, ...\right\}$$

W = {x : g'(x) = 0}

g(x) = cos (2π cos x) $\Rightarrow$ g'(x) = – sin (2π sin x) · (2π cos x) = 0

cos x = 0 $\Rightarrow$ x = (2n + 1) $\frac{\pi}{2}$

sin (2π sin x) = 0 $\Rightarrow$ 2π sin x = nπ $\Rightarrow$ sin x = $\frac{n}{2} = -1, \frac{-1}{2}, 0, \frac{1}{2}, 1$

$$W = \left\{\frac{n\pi}{2}, n\pi \pm \frac{\pi}{6}, n \in I\right\} = \left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, ...\right\}$$

Now, check the options

53. Answer the following by appropriately matching the lists based on the information given in the paragraph

Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y.

Suppose that another circle $C_3 : (x-h)^2 + (y-k)^2 = r^2$ satisfies the following conditions :

(i) centre of C_3 is collinear with the centres of C_1 and C_2,

(ii) C_1 and C_2 both lie inside C_3, and

(iii) C_3 touches C_1 at M and C_2 at N.

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.

There are some expression given in the List-I whose values are given in List-II below:

	List-I		**List-II**
(I)	$2h + k$	(P)	6
(II)	$\dfrac{\text{Length of ZW}}{\text{Length of XY}}$	(Q)	$\sqrt{6}$
(III)	$\dfrac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$	(R)	$\dfrac{5}{4}$
(IV)	α	(S)	$\dfrac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\dfrac{10}{3}$

Which of the following is the only CORRECT combination ?

Options

(1*) (II), (Q) (2) (I), (S) (3) (II), (T) (4) (I), (U)

Ans. 1

54. Answer the following by appropriately matching the lists based on the information given in the paragraph

Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y.

Suppose that another circle $C_3 : (x-h)^2 + (y-k)^2 = r^2$ satisfies the following conditions :

(i) centre of C_3 is collinear with the centres of C_1 and C_2,

(ii) C_1 and C_2 both lie inside C_3, and

(iii) C_3 touches C_1 at M and C_2 at N.

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.

There are some expression given in the List-I whose values are given in List-II below:

	List-I		List-II
(I)	$2h + k$	(P)	6
(II)	$\dfrac{\text{Length of ZW}}{\text{Length of XY}}$	(Q)	$\sqrt{6}$
(III)	$\dfrac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$	(R)	$\dfrac{5}{4}$
(IV)	α	(S)	$\dfrac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\dfrac{10}{3}$

Which of the following is the only INCORRECT combination ?

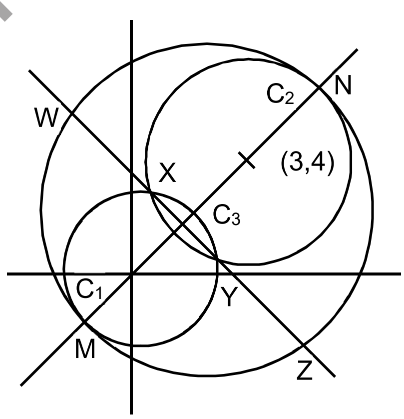
Options

(1) (III), (R) (2*) (IV), (S) (3) (IV), (U) (4) (I), (P)

Ans. 2

Sol. $MC_1 + C_1C_2 + C_2N = 2r$

$\Rightarrow 3 + 5 + 4 = 2r \Rightarrow r = 6 \Rightarrow$ Radius of $C_3 = 6$



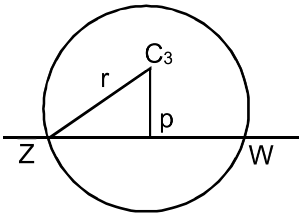
Suppose centre of C_3 be $(0 + r_4 \cos\theta, 0 + r_4 \sin\theta)$, $\left\{\begin{array}{l} r_4 = C_1C_3 = 3 \\ \tan\theta = \dfrac{4}{3} \end{array}\right\}$

$C_3 = \left(\dfrac{9}{5}, \dfrac{12}{5}\right) = (h, k) \Rightarrow 2h + k = 6$

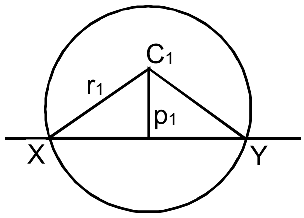
Equation of ZW and XY is $3x + 4y - 9 = 0$

(common chord of circle $C_1 = 0$ and $C_2 = 0$)

$ZW = \sqrt{r^2 - p^2} = \dfrac{24\sqrt{6}}{5}$ (where $r = 6$ and $p = \dfrac{6}{5}$)



$XY = 2\sqrt{r_1^2 - p_1^2} = \frac{24}{5}$ (where $r_1 = 3$ and $p_1 = \frac{9}{5}$)



$\frac{\text{Length of } ZW}{\text{Length of } XY} = \sqrt{6}$

Let length of perpendicular from M to ZW be λ, $\lambda = 3 + \frac{9}{5} = \frac{24}{5}$

$$= \frac{\text{Area of } \Delta MZN}{\text{Area of } \Delta ZMW} = \frac{\frac{1}{2}(MN) \times \frac{1}{2}(ZW)}{\frac{1}{2} \times ZW \times \lambda} = \frac{MN}{2\lambda} = \frac{5}{4}$$

$$C_3 : \left(x - \frac{9}{5}\right)^2 + \left(y - \frac{12}{5}\right)^2 = 6^2$$

$C_1 : x^2 + y^2 - 9 = 0$

common tangent to C_1 and C_3 is common chord of C_1 and C_3 is $3x + 4y + 15 = 0$.

Now, $3x + 4y + 15 = 0$ is tangent to parabola $x^2 = 8\alpha y$.

$$x^2 = 8\alpha\left(\frac{-3x - 15}{4}\right) \Rightarrow 4x^2 + 24\alpha x + 120\alpha = 0$$

$D = 0 \Rightarrow \alpha = \frac{10}{3}$.